Chapter III:
Interactions of electrons with matter

Contents of the chapter

- Electronic stopping force
- Nuclear stopping force
- Radiative collisions
- Cherenkov effect
- Electron trajectory

Basic considerations about e^- and e^+

- Large energy transfer possible
- large angular deviations possible → curled trajectory
- Incident positrons can transfer all their energy to a target electron in one collision ↔ $T_{max} = \gamma E = E$
- Incident electron and target electron are indistinguishable → After a collision the most energetic is followed (by convention) and $T_{max} = E/2$
- e^-/e^+ are « quickly » relativistic ($E_0 = m_e c^2 = 511$ keV)

Electronic stopping power for e$^-$ (1)

- As for incident ions → necessary to distinguish between distant and close collisions → or equivalently between collisions with large or small transferred energy *Q* (Q_0, as « boundary »)
- For distant collisions → same procedure as for Bethe equation →

$$-\frac{1}{\rho}\frac{dE^l_{elec}}{dx} = \frac{2\pi r_e^2 mc^2}{\beta^2}\frac{N_A}{M_u}\frac{Z}{A}\left[\ln\left(\frac{2mc^2\beta^2 Q}{(1-\beta^2)I}\right) - \beta^2\right]$$

- For close collisions → procedure of Møller (1932) taking into account relativistic effects, spin effects and exchange effect (electrons are indistinguishable) with $\tau = E/mc^2$ →

$$-\frac{1}{\rho}\frac{dE^p_{elec}}{dx} = \frac{2\pi r_e^2 mc^2}{\beta^2}\frac{N_A}{M_u}\frac{Z}{A}\frac{dQ}{Q^2} \times \left[1 + \frac{Q^2}{(E-Q)^2} + \frac{\tau^2}{(\tau+1)^2}\left(\frac{Q}{E}\right)^2 - \frac{(2\tau-1)}{(\tau+1)^2}\frac{Q}{(E-Q)}\right]$$

Electronic stopping power for e^- (2)

- By combining results for distant and close collisions and by including shell and density corrections →

$$-\frac{1}{\rho}\frac{dE_{elec}}{dx} = \frac{4\pi r_e^2 mc^2}{\beta^2}\frac{N_A}{M_u}\frac{Z}{A}\left[\ln\left(\frac{E}{I}\right) + \ln\left(1+\frac{\tau}{2}\right)^{1/2} + F^-(\tau) - \frac{\delta}{2} - \frac{C}{Z}\right]$$

with

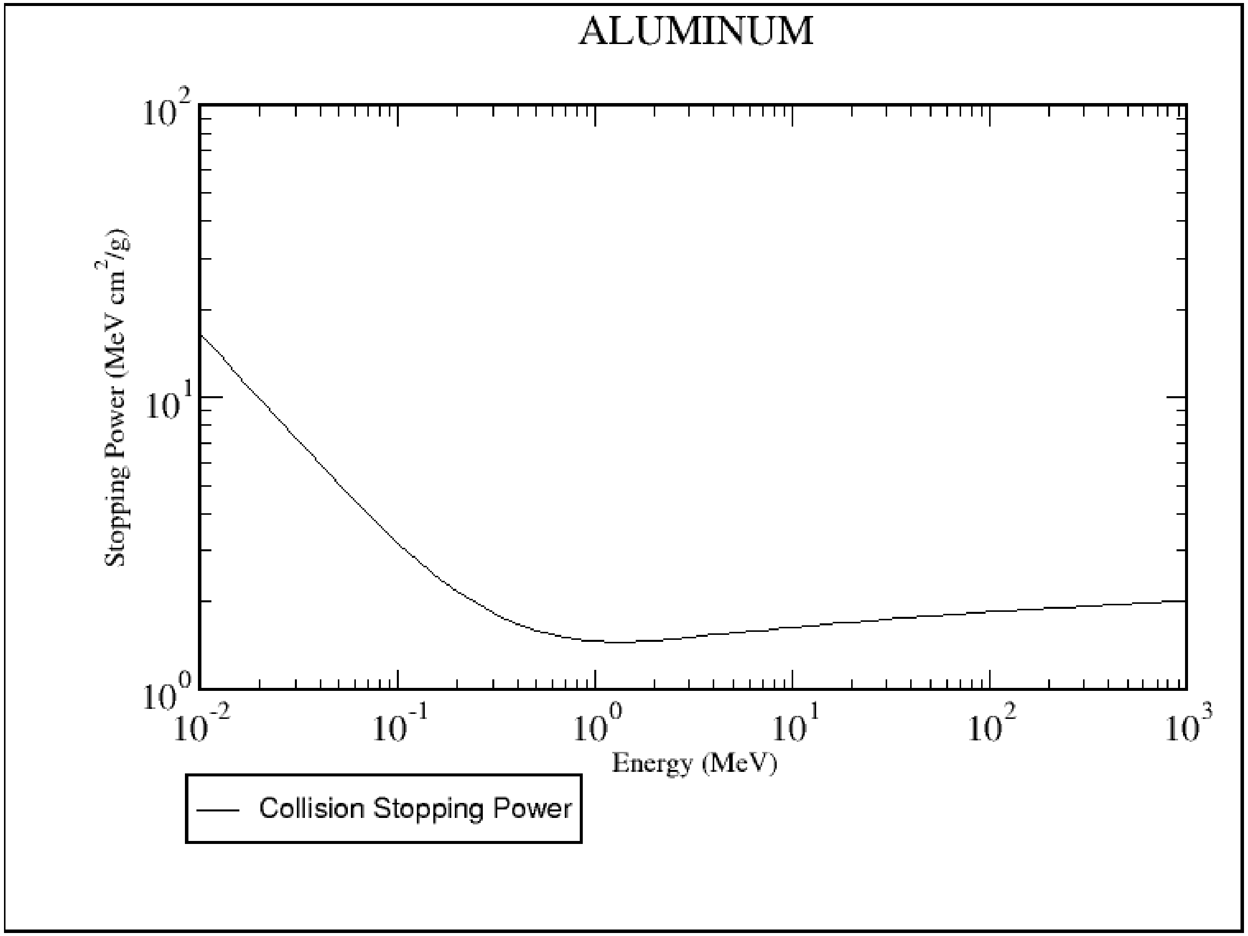
$$F^-(\tau) = \frac{1-\beta^2}{2}\left[1 + \frac{\tau^2}{8} - (2\tau+1)\ln 2\right]$$

- We note that the first factor is the same as for ions

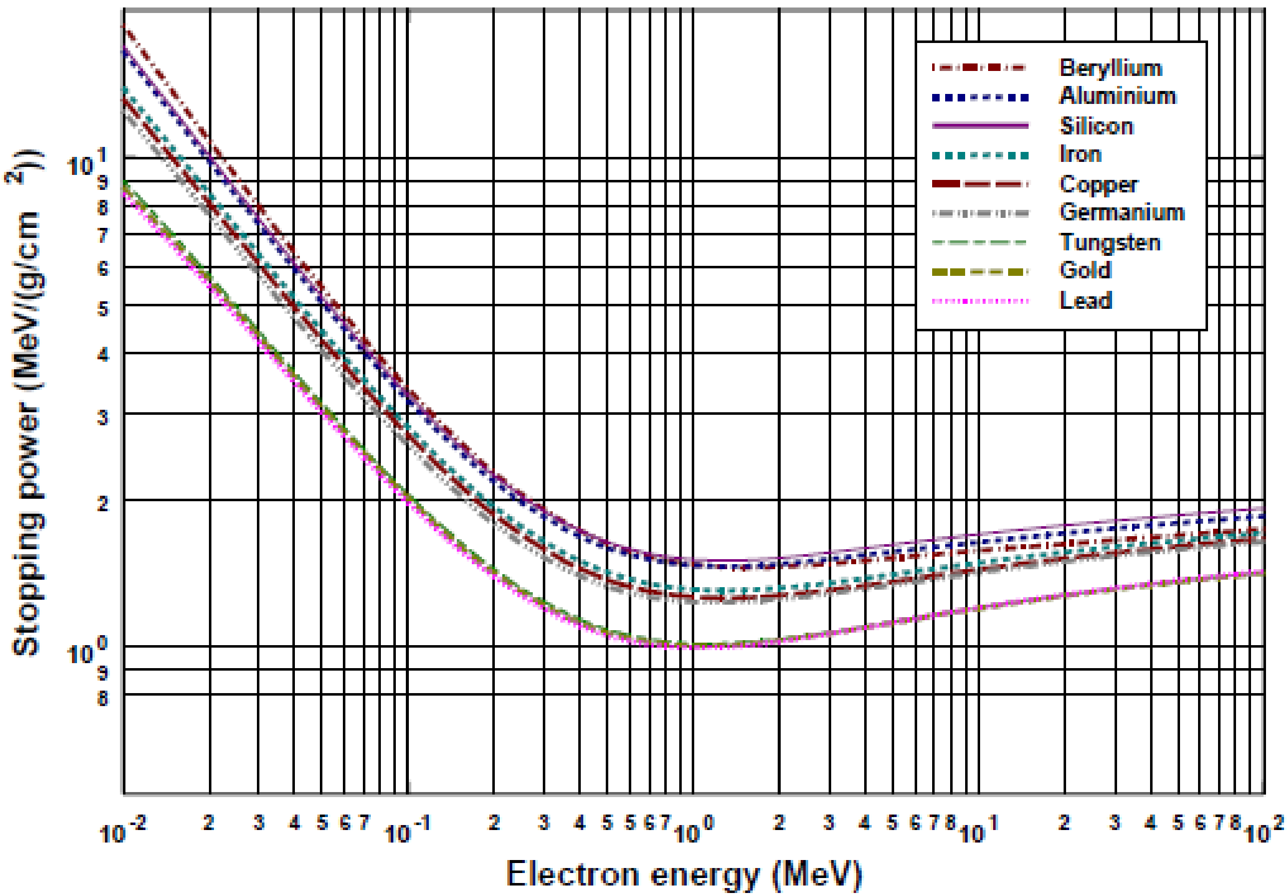
Example of electronic stopping for e^-

e^- incident on aluminium

www.nist.gov/pml/data/star/index.cfm



Electronic stopping for e^- in ≠ media



- *dE/dx* ≈ constant for *E* > 1 MeV
- Weak difference between all media

Electronic stopping power for e^+

- Same expression as for electrons with F^- replaced by F^+ (taking into account that all kinetic energy can be transferred into only one collision) →

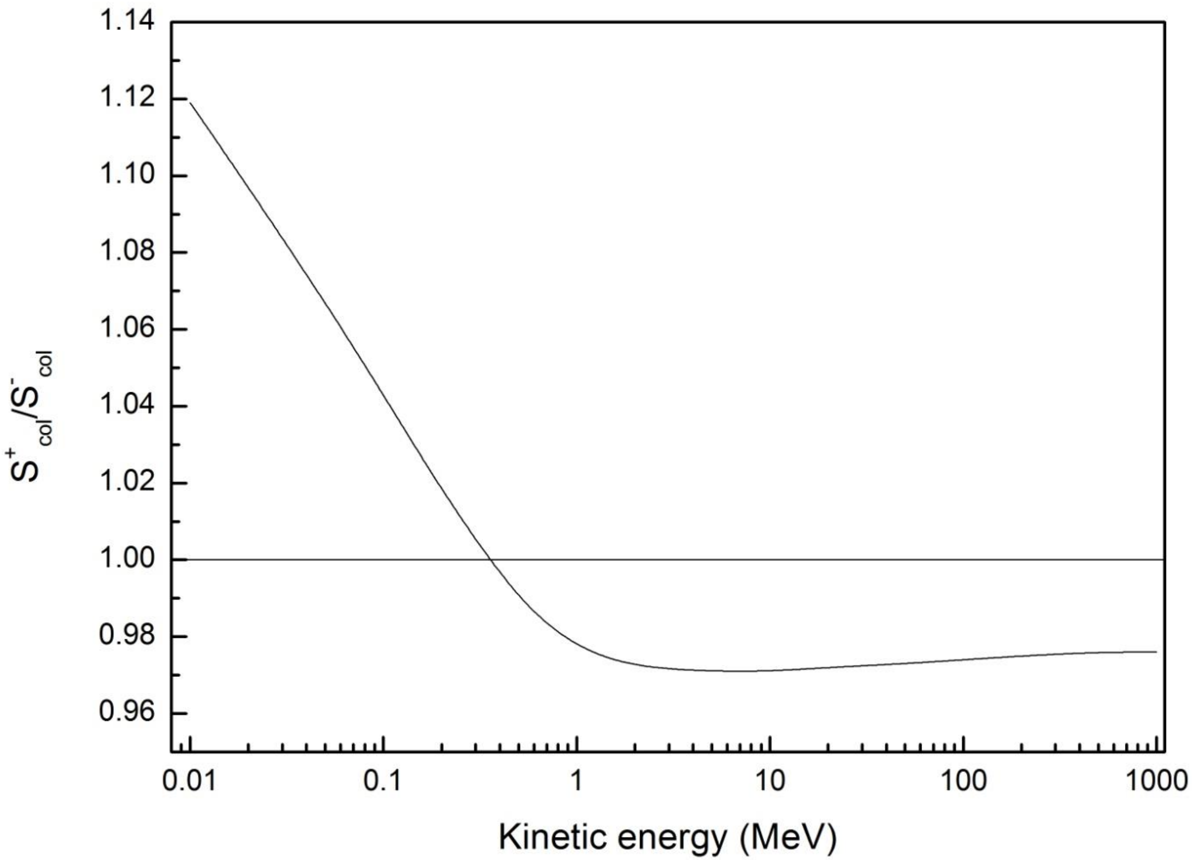
$$-\frac{1}{\rho}\frac{dE_{elec}}{dx} = \frac{4\pi r_e^2 mc^2}{\beta^2}\frac{N_A}{M_u}\frac{Z}{A}\left[\ln\left(\frac{E}{I}\right) + \ln\left(1+\frac{\tau}{2}\right)^{1/2} + F^+(\tau) - \frac{\delta}{2} - \frac{C}{Z}\right]$$

with

$$F^+(\tau) = \ln 2 - \frac{\beta^2}{24}\left[23 + \frac{14}{\tau+2} + \frac{10}{(\tau+2)^2 + \frac{4}{(\tau+2)^3}}\right]$$

Comparison between e^- and e^+

e^- and e^+ incident on aluminium



ICRU Report 37

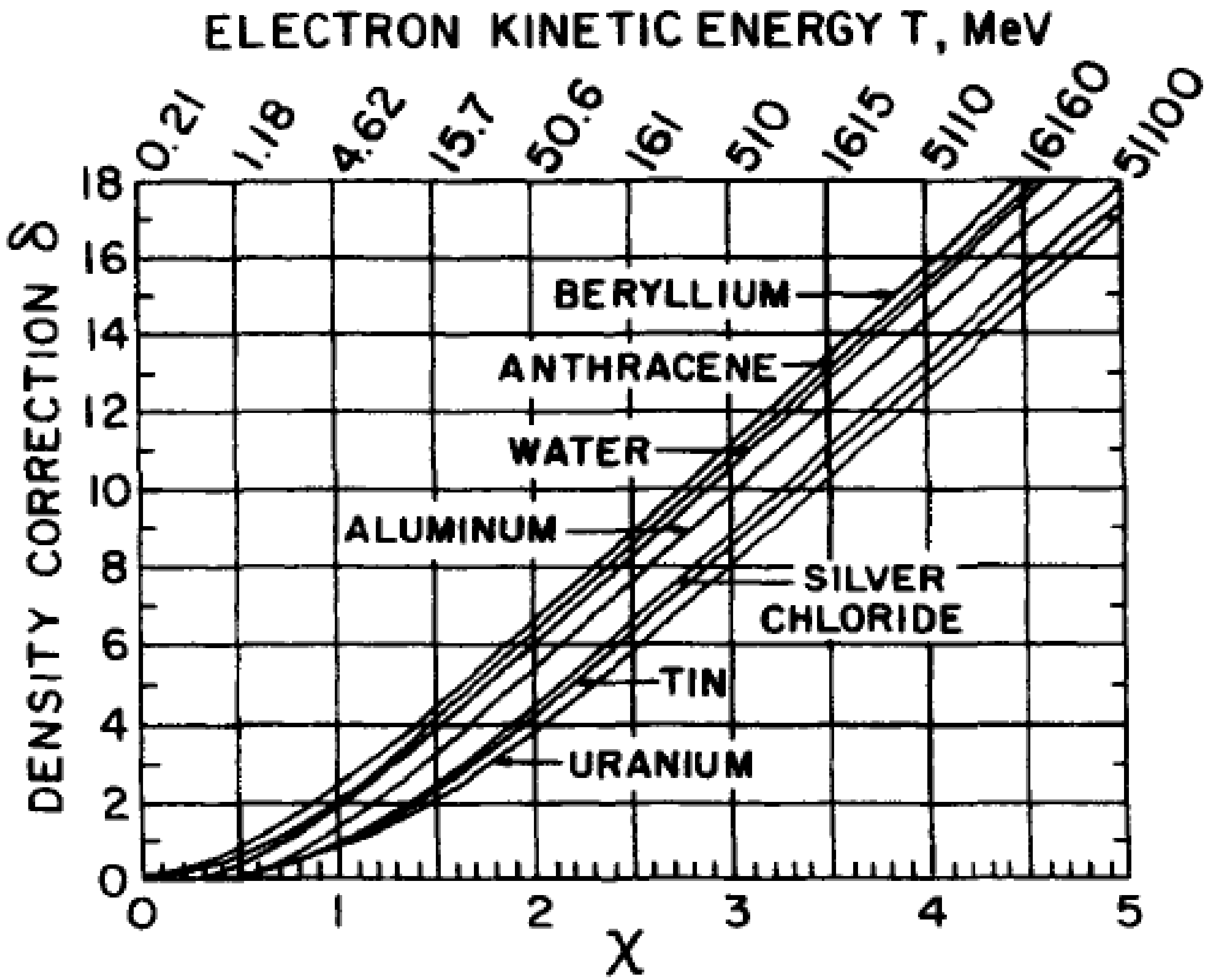
Remark about density effect (δ) for e^- and e^+

- For ion → density effect significant for large energies
- For electron (with small mass) → density effect becomes important for smaller energies → must be considered for electrons emitted during nuclear disintegrations
- Complete study made by Sternheimer (1952) → δ depends on the composition and on the density of the medium as well as on the velocity of the particle via the χ parameter →

$$\chi = \log \frac{p}{mc} = \log \beta\gamma_1$$

- We notice that δ ↗ when χ ↗ (pour $\chi > 1$) and becomes « consequent » for E > 511 keV (χ = 0.24) and δ ↘ when Z ↗ (media with small Z are more efficiently polarized) → important for biologic media

Density effect for e^- and e^+ (1)



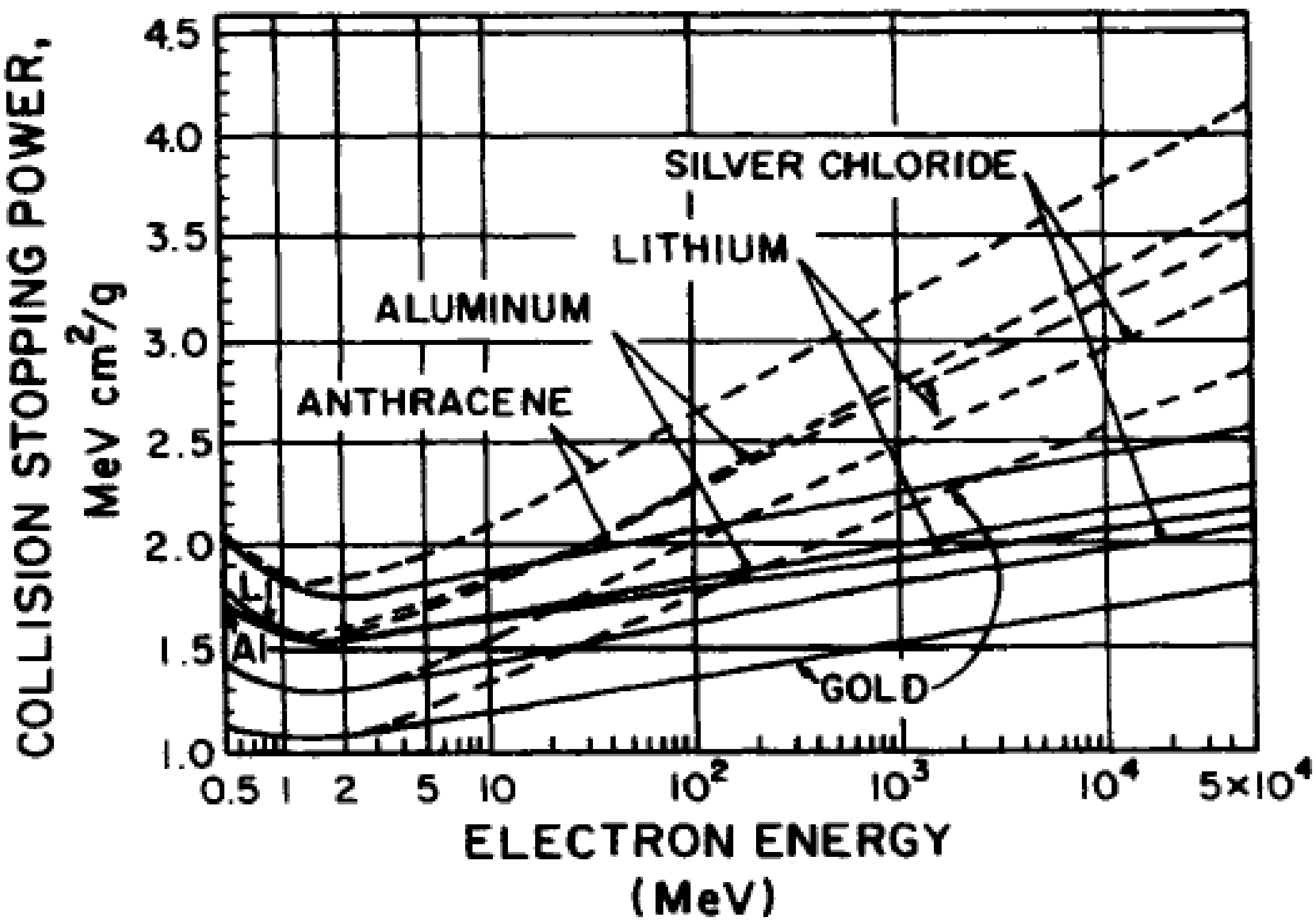
Density effect for e^- and e^+ (2)

Polarization Effect for Electrons

T (MeV)	Effect[a] (%)		
	C	Cu	Au
0.1	0	0	0
1.0	3	1.5	0.7
5	9	7	5
10	12	10	8
50	20	18	15

[a] Decrease in mass collision stopping power for condensed media vs. gases.

Density effect for e^- and e^+ (3)



Restricted stopping power (linear energy transfer)

L_Δ: **Linear energy transfer** (LET) (or Restricted stopping power) →

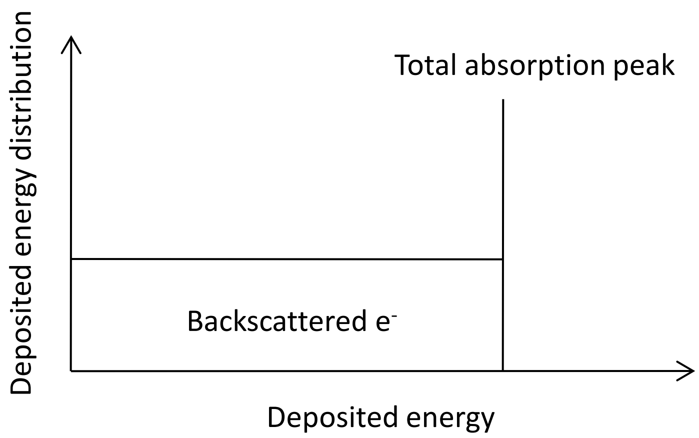
$$L_\Delta = \frac{dE_\Delta}{dx}$$
$$L_\Delta = \frac{dE_{elec}}{dx} - \frac{dE_{KE>\Delta}}{dx}$$

with $dE_\Delta = dE_{elec} - dE_{KE>\Delta}$ et $dE_{KE>\Delta}$: sum of kinetic energies for secondary e^- (e^- δ) with kinetic energy > the Δ energy → dE_Δ is the locally transferred energy

L_∞: **Non-restricted stopping power** → $L_\infty = \frac{dE_{elec}}{dx}$

Nuclear stopping for e^- and e^+

- Collision with nuclei do not give any contribution to stopping
- These collisions explain the curved trajectory of electrons in matter
- In general: large number of small deviation collisions
- Small probability of very large deviation (up to 180°)
- Electron backscattering possible → deposited energy in matter:



Radiative collisions (1)

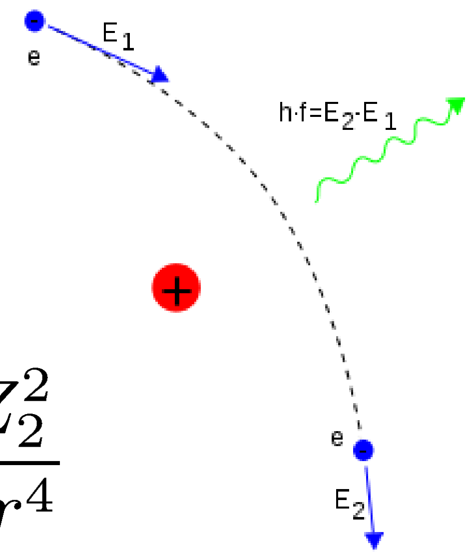
- A free charged particle (+ or -) accelerating → energy dissipated by electromagnetic radiation
- Radiation called deceleration radiation or Bremsstrahlung
- For $v \ll c$ → the radiated power P is given the equation of Larmor (see electromagnetism teaching) →

$$P = \frac{2}{3}\frac{e_1^2}{4\pi\epsilon_0 c^3}a^2$$

with ϵ_0 the vacuum permittivity, e_1 the charge of the particle and *a,* its acceleration

Radiative collisions (2)

- The radiated power is proportional to the square of the acceleration → if we consider the force F between an incident particle 1 (energy E, charge z_1e and mass m_1) and a target particle 2 (charge Z_2e) →



$$F = \frac{z_1 Z_2 e^2}{4\pi\epsilon_0 r^2} = m_1 a \Rightarrow P \propto z_1^2 a^2 \propto \frac{z_1^4 Z_2^2}{m_1^2 r^4}$$

Characteristics of Bremsstrahlung

- $P \propto m_1^{-2}$ → radiative process negligible for incident ions → must be considered only for incident e^- and e^+
- The acceleration can happen in the field of the nucleus (n) or of an atomic electron (e) → weaker because of charge *= e* → but as Z_2 electrons are present → the total cross section will be multiplied by Z_2 (global effect) → at the end difference of a factor Z_2
- The mass radiative stopping power is written→

$$-\frac{1}{\rho}\frac{dE_{rad}}{dx} = \frac{N_A}{M_u A}\left[\int h\nu \frac{d\sigma_n}{d(h\nu)} d(h\nu) + Z_2 \int h\nu \frac{d\sigma_e}{d(h\nu)} d(h\nu)\right]$$

with $d\sigma_{n,e}/d(h\nu)$, the differential cross sections for the emission of a photon with energy $h\nu$ due to the interactions with a nucleus or with an atomic electron

Characteristics of Bremsstrahlung for incident e⁻ (1)

- For an electron-ion interaction → energy emission spectrum for the photon continuous between 0 and *E*
- For an electron-electron interaction → energy emission spectrum for the photon continuous between 0 and *E′* with (energy conservation + indistinguishable e⁻ + relativistic corrections) →

$$E' = mc^2 E[E + 2mc^2 - \beta(E + mc^2)]^{-1}$$

- Usually non-dimensional cross sections for radiative energy losses are introduced →

$$\Phi_{rad,n} = (\alpha r_e^2 Z_2^2)^{-1} \int_0^E (h\nu / E_{tot}) \frac{d\sigma_n}{d(h\nu)} dh\nu$$

$$\Phi_{rad,e} = (\alpha r_e^2)^{-1} \int_0^{E'} (h\nu / E_{tot}) \frac{d\sigma_e}{d(h\nu)} dh\nu$$

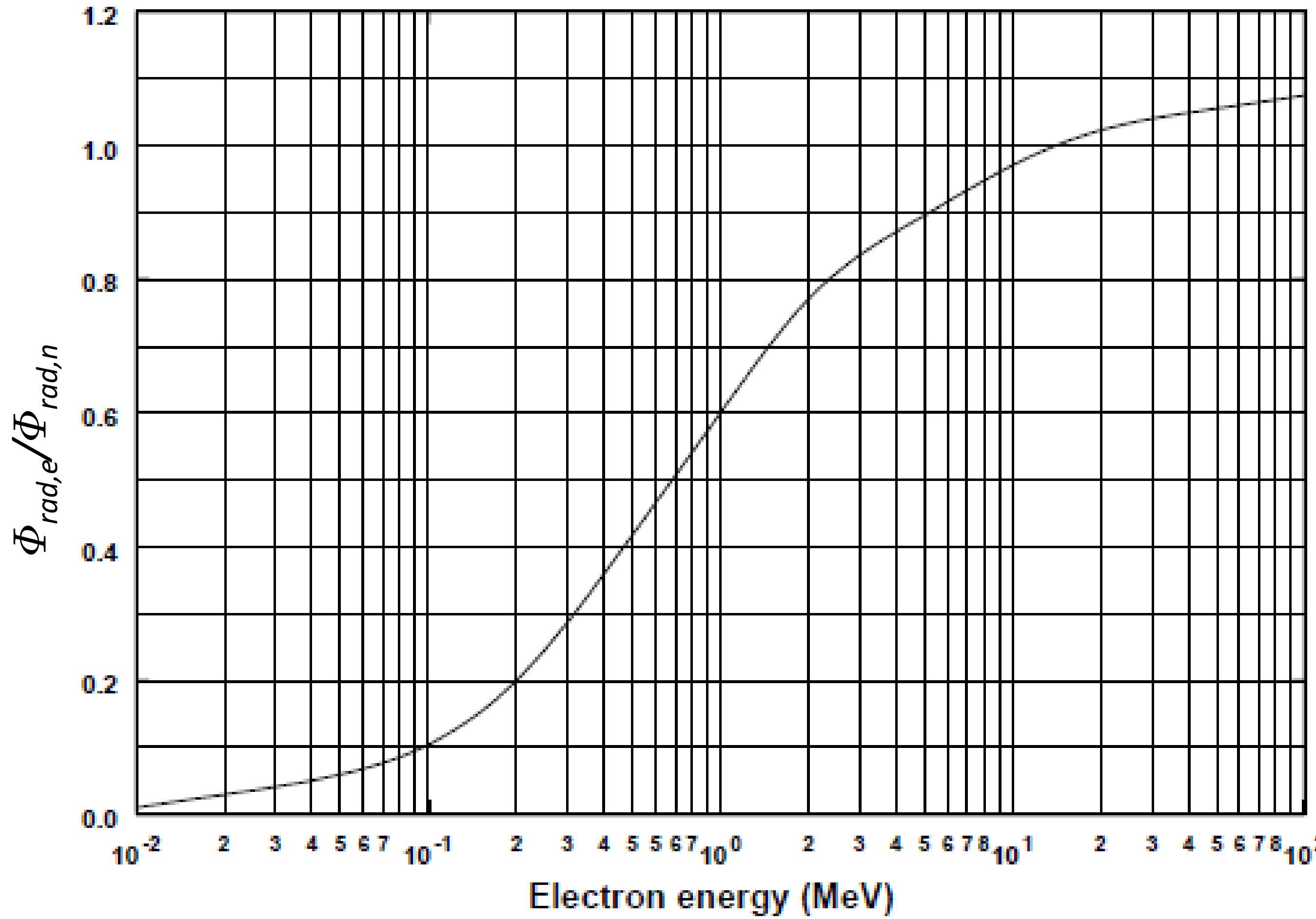
Characteristics of Bremsstrahlung for incident e⁻ (2)

- We introduce in previous expressions the constant of fine structure α = 1/137.036 and $E_{tot} = E + mc^2$, the total energy of the electron
- With these notations → the mass radiative stopping power becomes →

$$-\frac{1}{\rho}\frac{dE_{rad}}{dx} = \frac{N_A}{M_u A}\alpha r_e^2 E_{tot} Z_2^2 \Phi_{rad,n}[1 + (1/Z)\Phi_{rad,e}/\Phi_{rad,n}]$$

- Usually → $\Phi_{rad,e}/\Phi_{rad,n}$ is supposed to be equal to 1, but it is only true at high energies → for E ↘ → it is possible to show that it tends towards 0

$\Phi_{rad,e}/\Phi_{rad,n}$ ratio



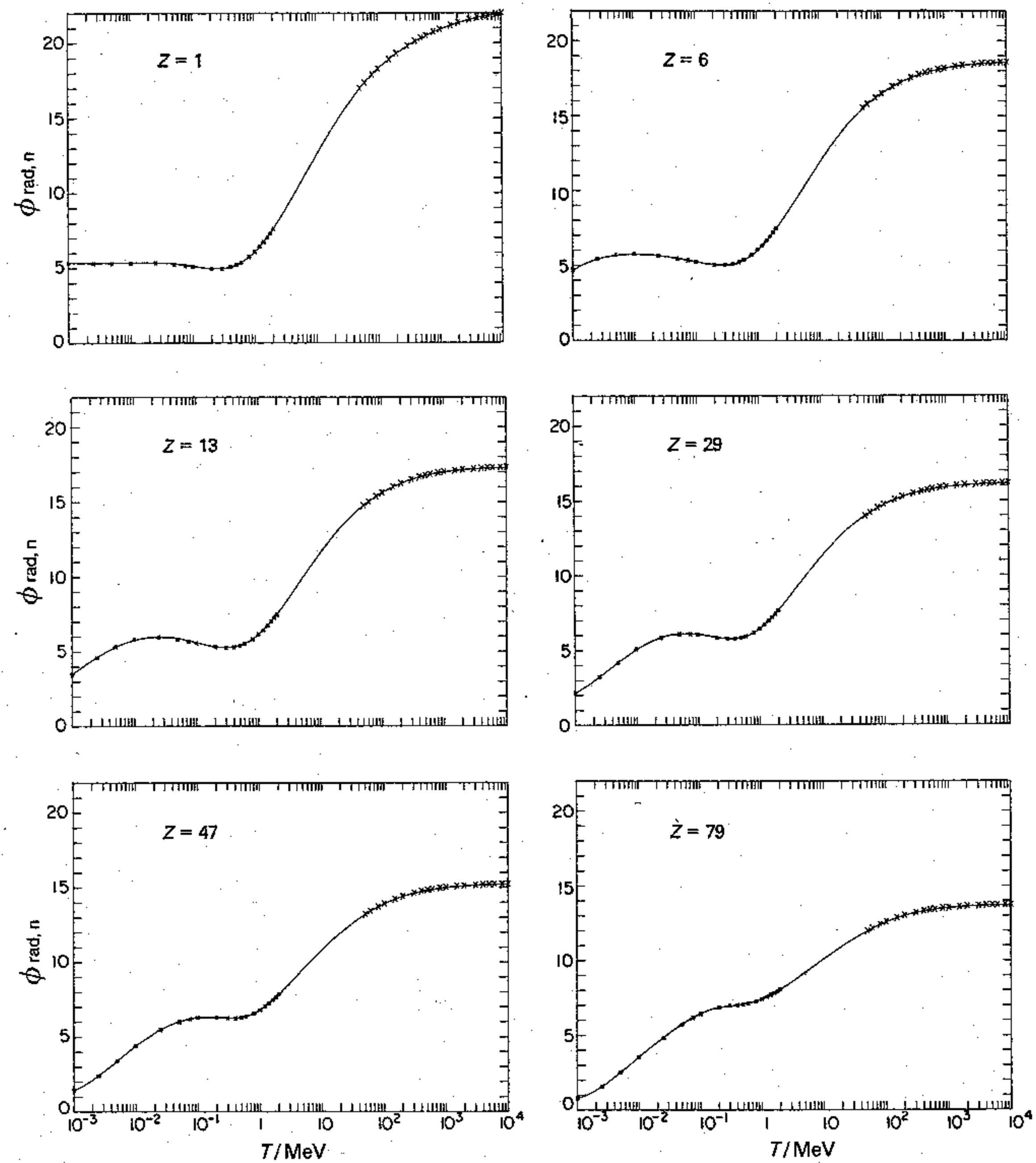
$\Phi_{rad,e}/\Phi_{rad,n}$ does not depend much on *Z*

Determination of $\Phi_{rad,n}$

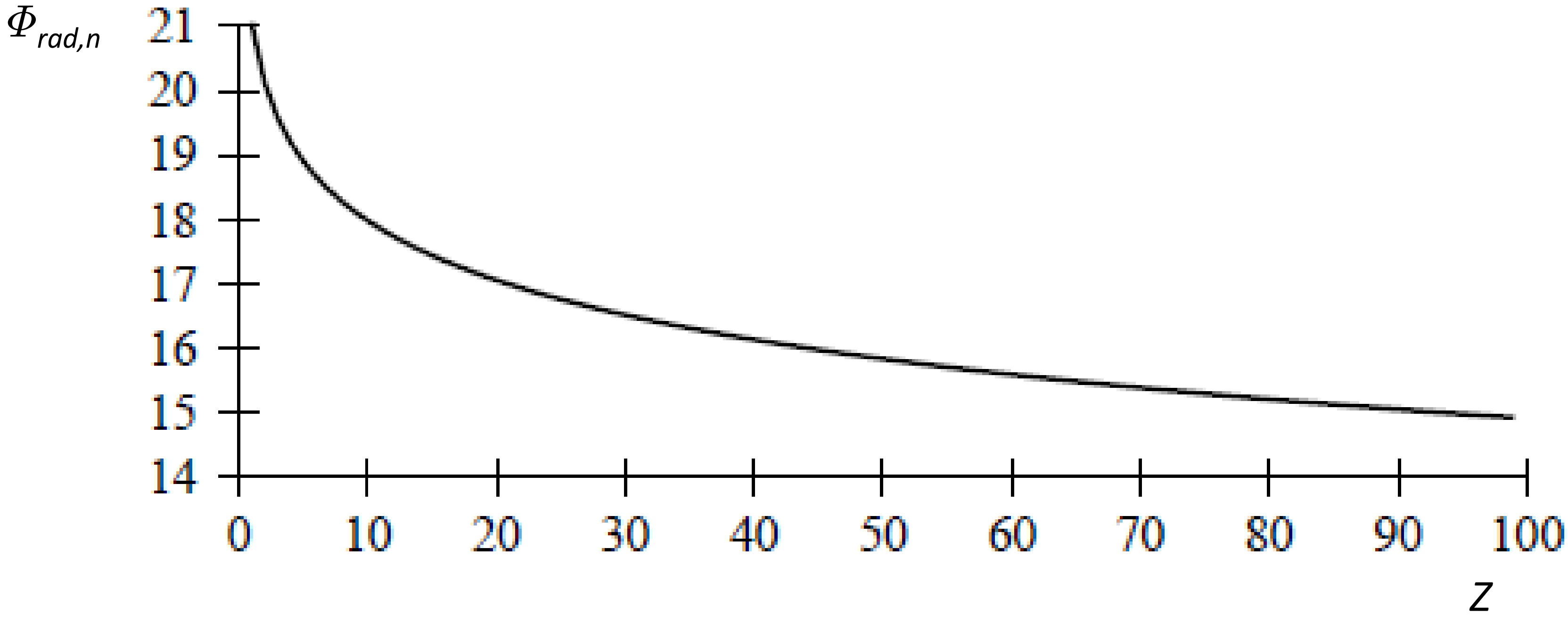
- Complex calculation → ≠ approximations for high E ($E > 50$ MeV) and small energies ($E < 2$ MeV) → between these 2 limits → interpolation
- Moreover it is necessary to consider the screening of the nucleus by atomic electrons
- Finally $\Phi_{rad,n}$ is a function that slowly varies as a function of E and Z_2
- For $E < 2$ MeV → $\Phi_{rad,n} \approx 16/3$ (that can be obtained from a non- relativistic calculation) → constant cross section
- For high E → $\Phi_{rad,n}$ ↗ for E ↗ and tends towards →

$$\Phi_{rad,n} \rightarrow 4\left(\frac{1}{18} + \ln 183 Z_2^{-1/3}\right)$$

Examples of $\Phi_{rad,n}$ for various media



Asymptotic behaviour of $\Phi_{rad,n}$ as a function of Z

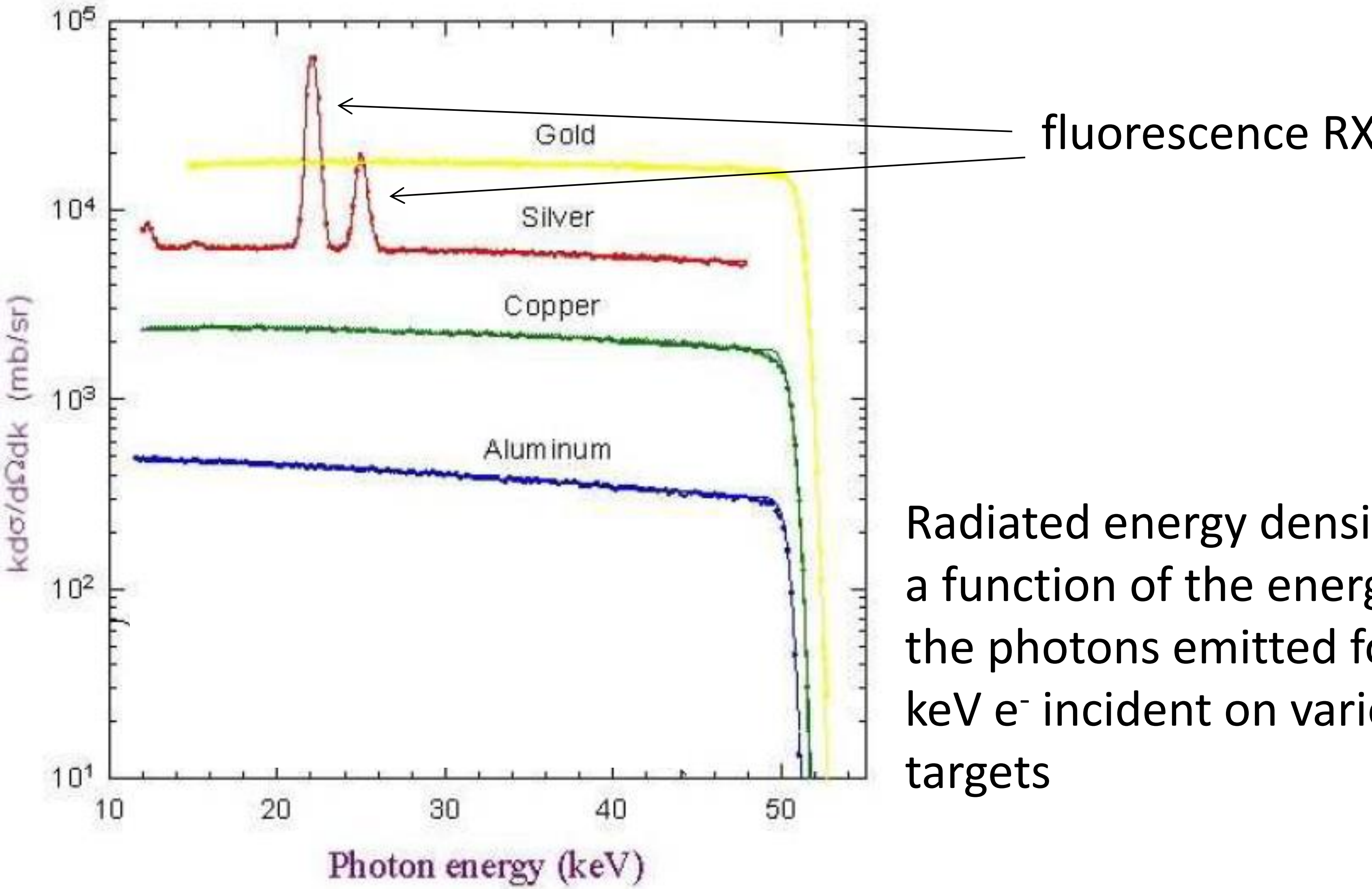


Differential cross sections

- It is possible to show that $h\nu d\sigma_n/d(h\nu)$ is independent on $h\nu$ for small energies of incident e^- → radiated energy density is constant
- For larger energies of incident e^- → $h\nu d\sigma_n/d(h\nu)$ ↘ when $h\nu$ ↗

- $d\sigma_n /d(\Omega)$ has a maximum $\perp$ to the direction of incident beam for small energies of incident e^-
- For larger energies → the maximum gradually moves to the direction of incident beam

Radiated energy density for thin targets

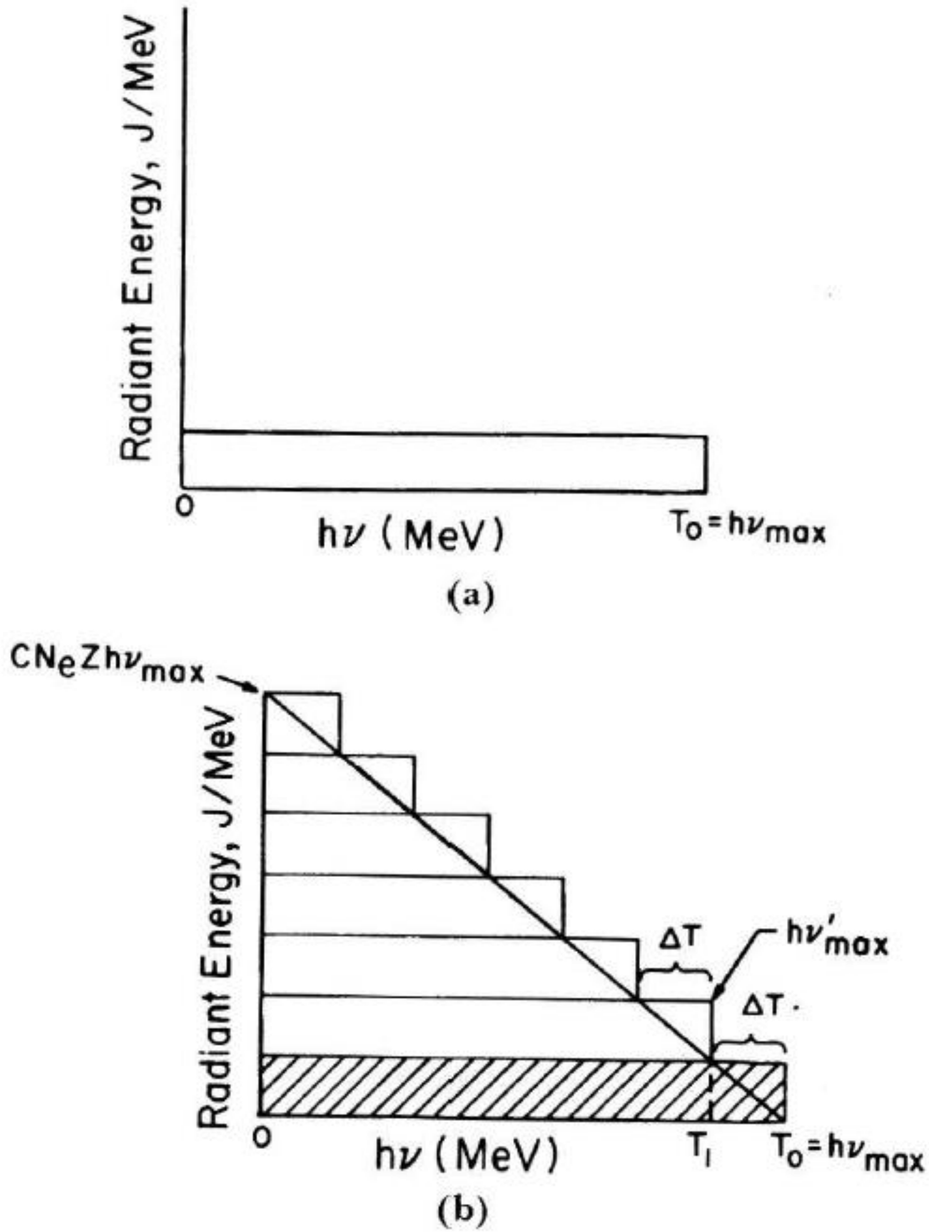


Radiated energy density as a function of the energy of the photons emitted for 60 keV e^- incident on various targets

Bremsstrahlung: thin target ↔ thick targets (1)

- In a thin target Bremsstrahlung is the photon emission in only one collision between electron and atom → process described by the differential cross section
- Bremsstrahlung in a thick target results from multiple interactions process of the electron → e^- loses an important part of its energy (or all its energy) in the target
- Radiated energy density in a thick target is thus the sum of radiated energy densities in a thin target for different energies

Bremsstrahlung: thin target ↔ thick targets (2)



Characteristics of Bremsstrahlung for incident e^+

- For incident e^+ → $(\Phi_{rad,n})^+ \approx (\Phi_{rad,n})^-$ for high energies but for small energies → $(\Phi_{rad,n})^+ < (\Phi_{rad,n})^-$ (absence of electric dipolar moment)

- Moreover for high E → $(\Phi_{rad,e})^+ \approx (\Phi_{rad,e})^-$ but for small E → $(\Phi_{rad,e})^+ > (\Phi_{rad,e})^-$

- Finally → radiative stopping power smaller for e^+ than for e^- at small incident energies and about equal for high energies

$\Phi_{rad,n}$ for incident e⁻ and e⁺

(T/Z^2)/MeV	$[\phi_{\mathrm{rad,n}}]^+/[\phi_{\mathrm{rad,n}}]^-$
1×10^{-7}	0.014
2×10^{-7}	0.030
5×10^{-7}	0.059
1×10^{-6}	0.087
1.18×10^{-6}	0.094*
2×10^{-6}	0.119
5×10^{-6}	0.166
5.91×10^{-6}	0.175*
1×10^{-5}	0.206
2×10^{-5}	0.253
5×10^{-5}	0.335
5.91×10^{-5}	0.359*
1×10^{-4}	0.415
1.56×10^{-4}	0.465*
2×10^{-4}	0.507
5×10^{-4}	0.640
7.81×10^{-4}	0.708*
1×10^{-3}	0.740
2×10^{-3}	0.816
5×10^{-3}	0.887
7.81×10^{-3}	0.916*
1×10^{-2}	0.928
2×10^{-2}	0.962
5×10^{-2}	0.991
1×10^{-1}	1.000

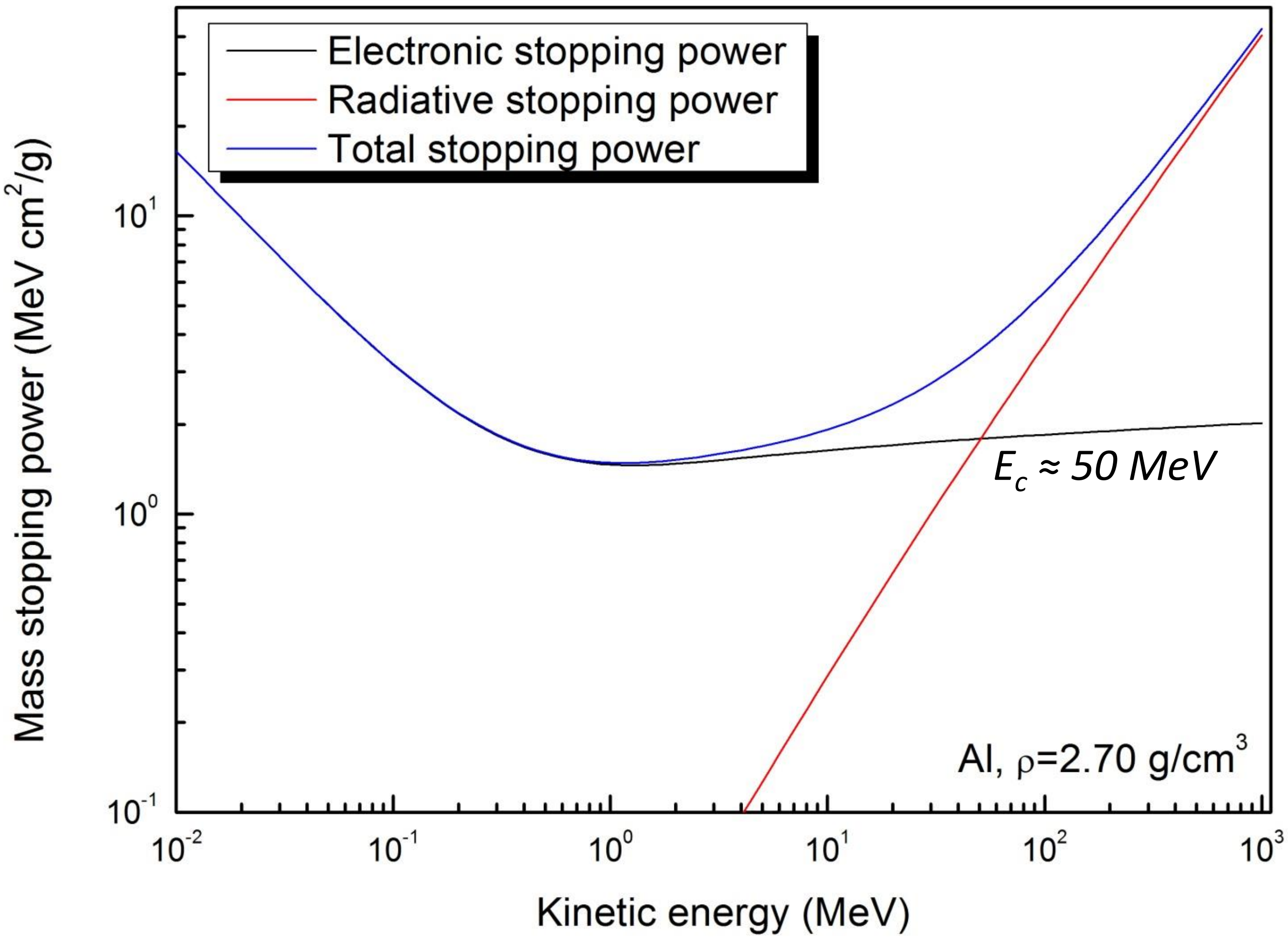
Total stopping power for electrons

- Total stopping power is the sum of electronic and radiative stopping powers (nuclear stopping power is negligible)
- As dE_{elec}/dx → constant when E ↗ and as $dE_{rad}/dx \propto E$ → For E ↗ → radiative losses become dominant
- As $dE_{elec}/dx \propto Z/A$ *et* $dE_{rad}/dx \propto Z^2/A$ → radiative stopping power increases more quickly with Z than the electronic stopping power
- The critical kinetic energy E_c for which both stoppings are equal ↘ when Z ↗
- $1/E_c$ linearly varies with Z → For E_c in MeV:

$$E_c = \frac{817}{Z + 1.97}$$

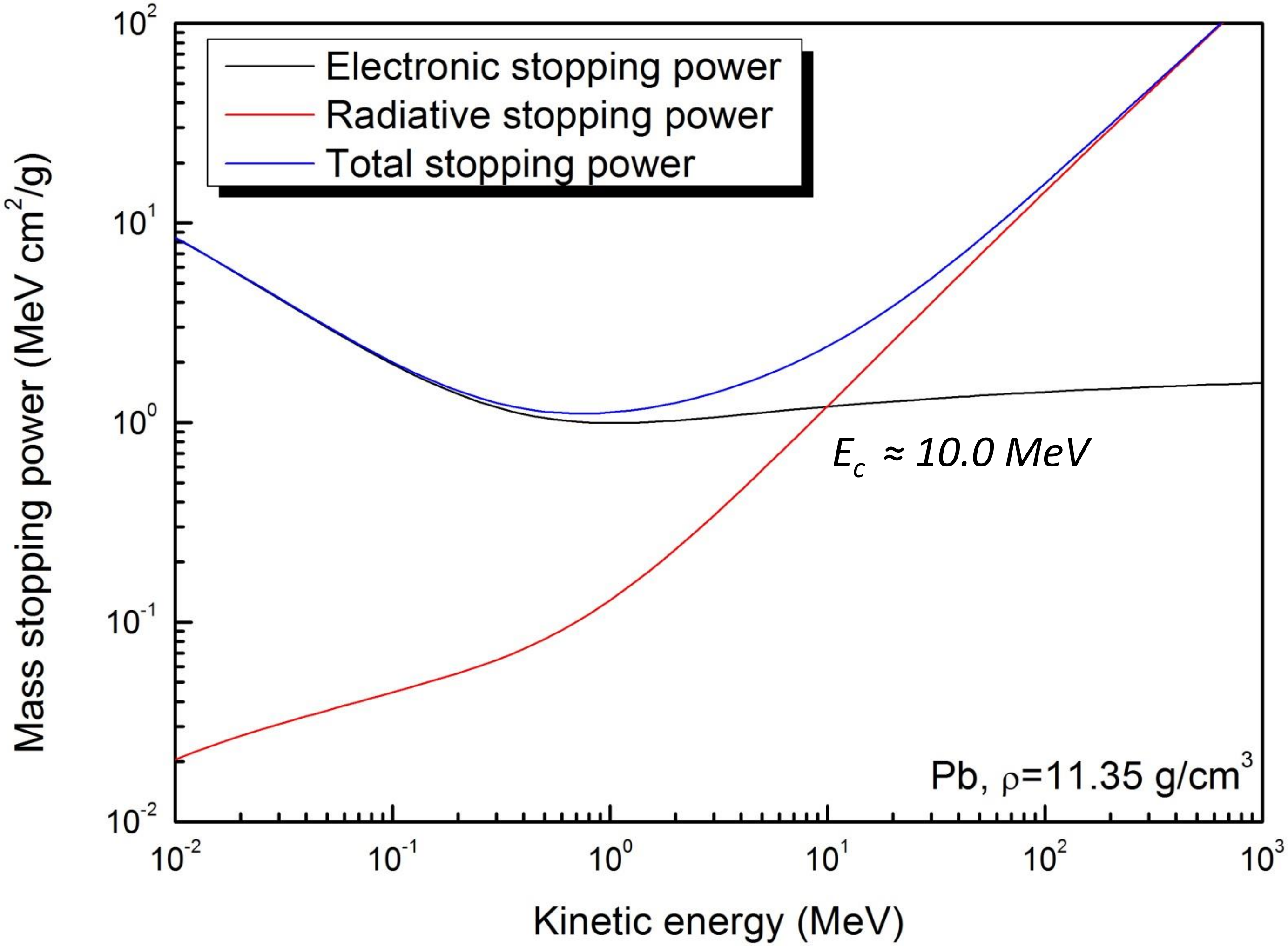
Stopping power for electron: example 1

http://physics.nist.gov/PhysRefData/Star/Text/ESTAR.html



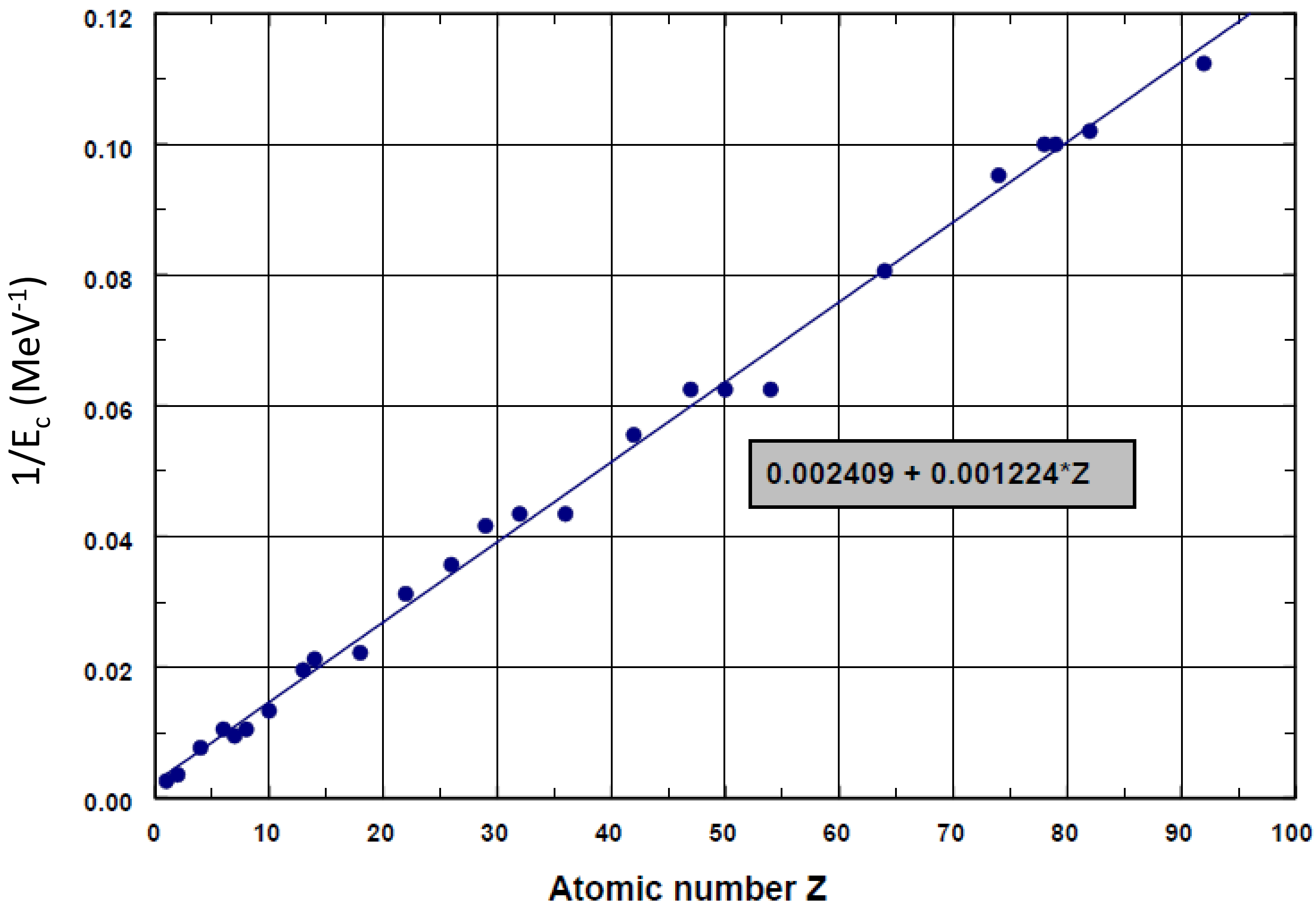
Incident electrons on Al (Z = 13) → with equation: E_c = 54.6 MeV

Stopping power for electron: example 2



Incident electrons on Pb (Z = 82) → with equation: E_c = 9.7 MeV

Evolution of E_c as a function of Z



Radiation yield (1)

- The radiation yield $Y(E_0)$ of an electron with initial kinetic energy E_0 is the fraction of energy emitted as photons for a complete stopping of the incident electron in the medium

- The fraction $y(E)$ of loss energy per unit of travelled distance that is converted into photons is given by

$$y(E) = \frac{dE_{rad}/dx}{dE_{tot}/dx} = \frac{dE_{rad}}{dE_{tot}}$$

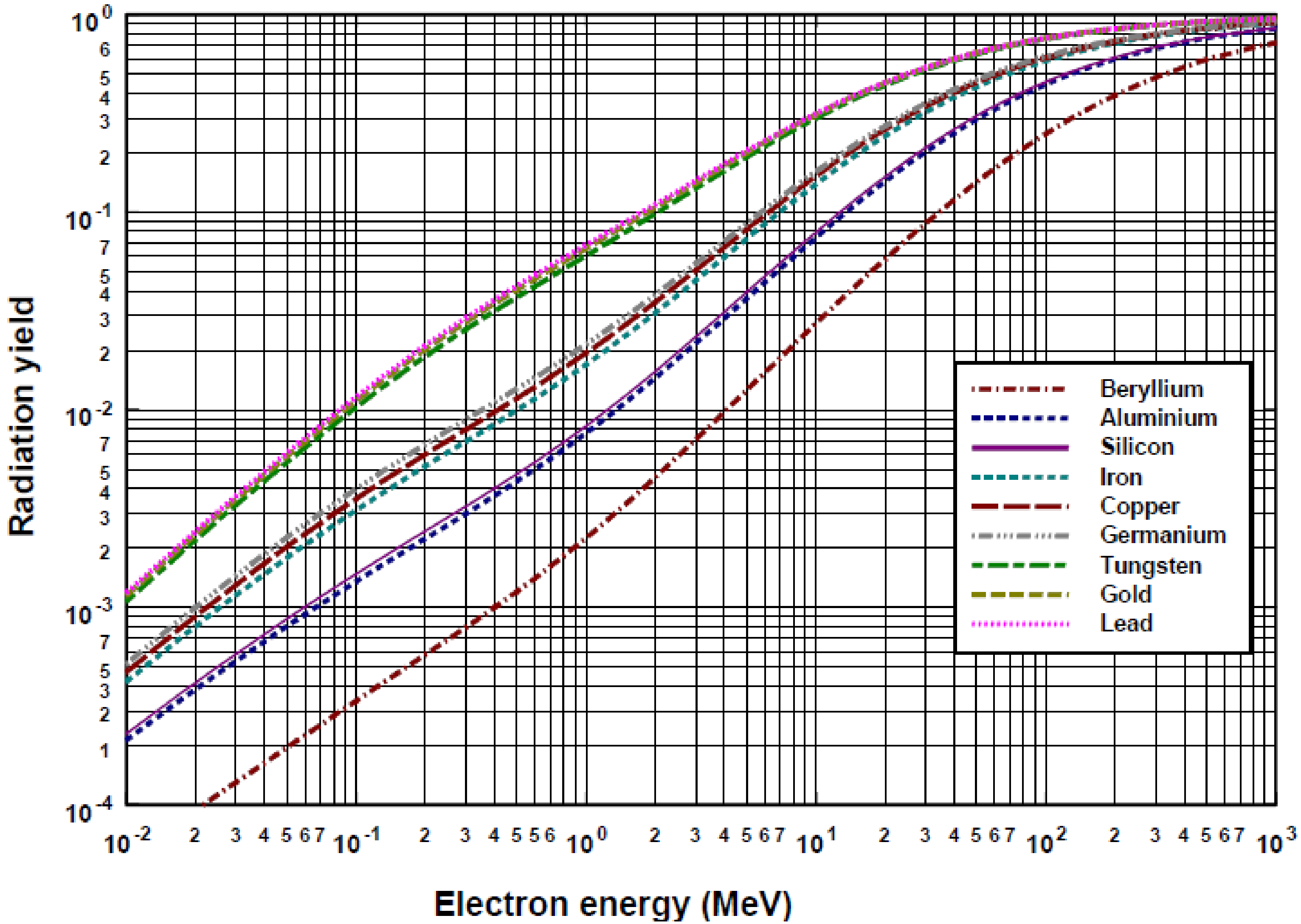
- Thus $Y(E_0)$ for an electron with initial energy E_0 is the mean value of $y(E)$ for E varying from E_0 to 0

$$Y(E_0) = \frac{\int_{parcours} dE_{rad}}{E_0} = \frac{1}{E_0} \int_0^{E_0} y(E) dE$$

Radiation yield (2)

- The radiation yield ↗ for E ↗ and Z ↗
- For small energies→ radiation yield is weak→ almost all e^- energy is dissipated as heat → target must be cooled

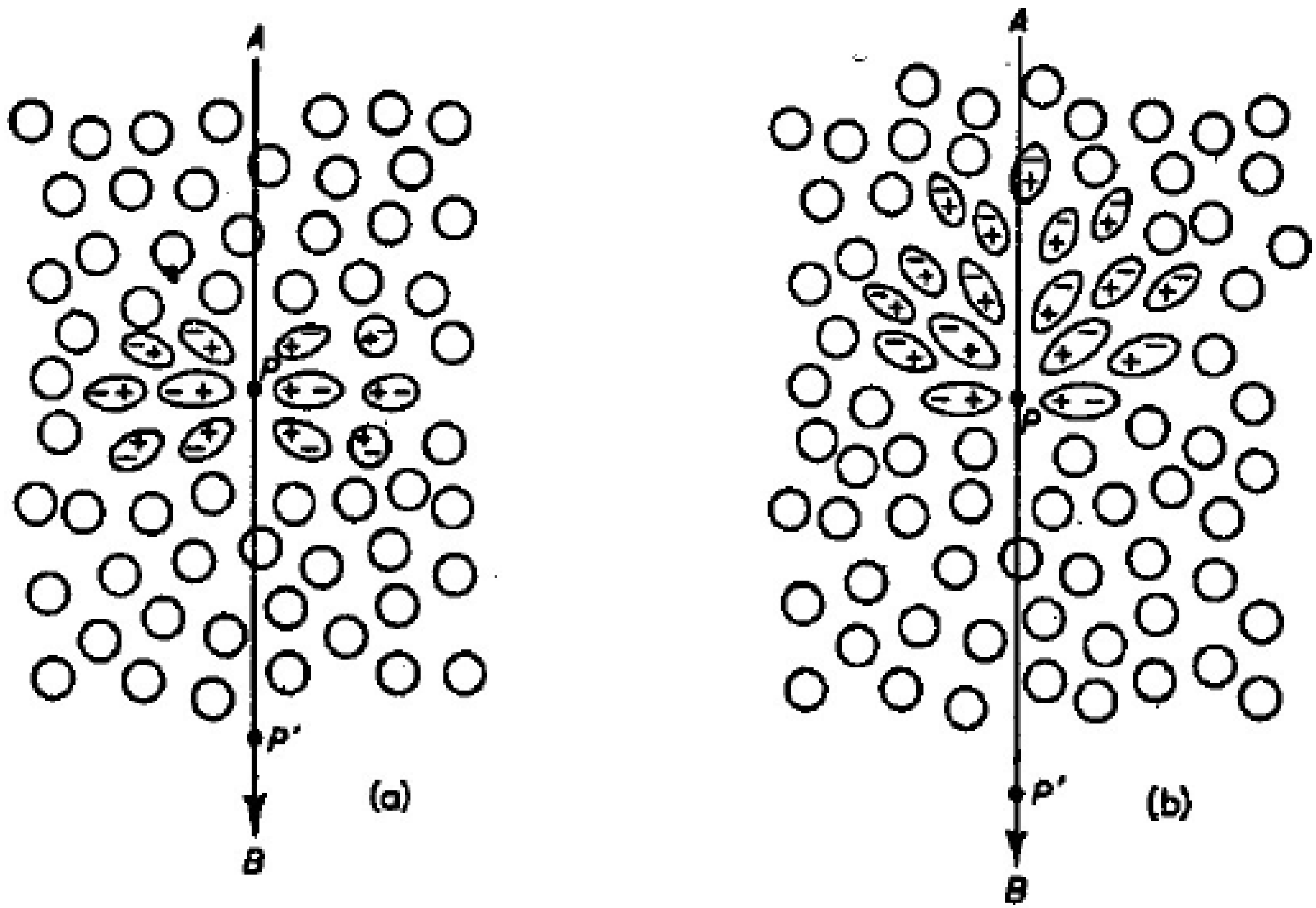
Radiation yield: Example



Cherenkov effect

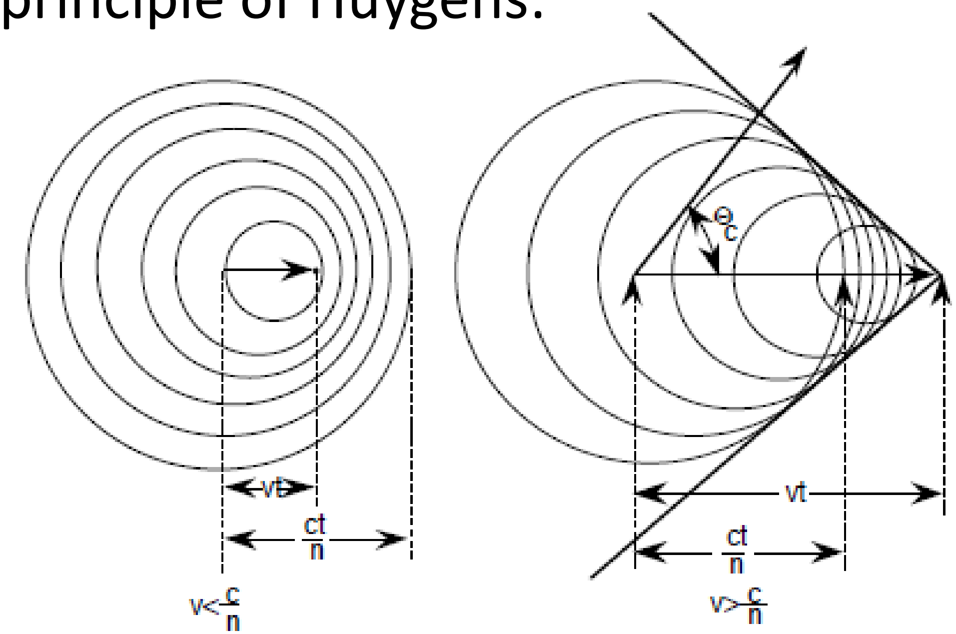
- When a charged particle travels in a medium faster than the light velocity in the medium (c/n with n refractive index of the medium) → radiation emission
- Phenomenon analog to the shock wave produced in air at supersonic velocities
- The particle polarizes the medium → for particle velocities < *c/n* → the dipoles are distributed in a symmetric way along the particle trajectory (in particular with respect to the plane ⊥ to the trajectory → net dipolar momentum equal to 0 → during the return to non-polarized state → aleatory electromagnetic perturbations (propagating with velocity *c/n*) that cancel themselves
- For particle velocities > *c/n* → the velocity for dipoles creation < particle velocity→ asymmetry with respect to the plane ⊥ to the trajectory → net dipolar momentum different from 0 → perturbations constructively interfere → apparition of a wave

Medium polarization by charged particle



Huygens construction for Cherenkov effect

- From the principle of Huygens:



- Direction of emission: $\cos\Theta_c = \frac{(c/n)t}{vt} = \frac{c}{nv} = \frac{1}{n\beta}$

Remarks on Cherenkov effect

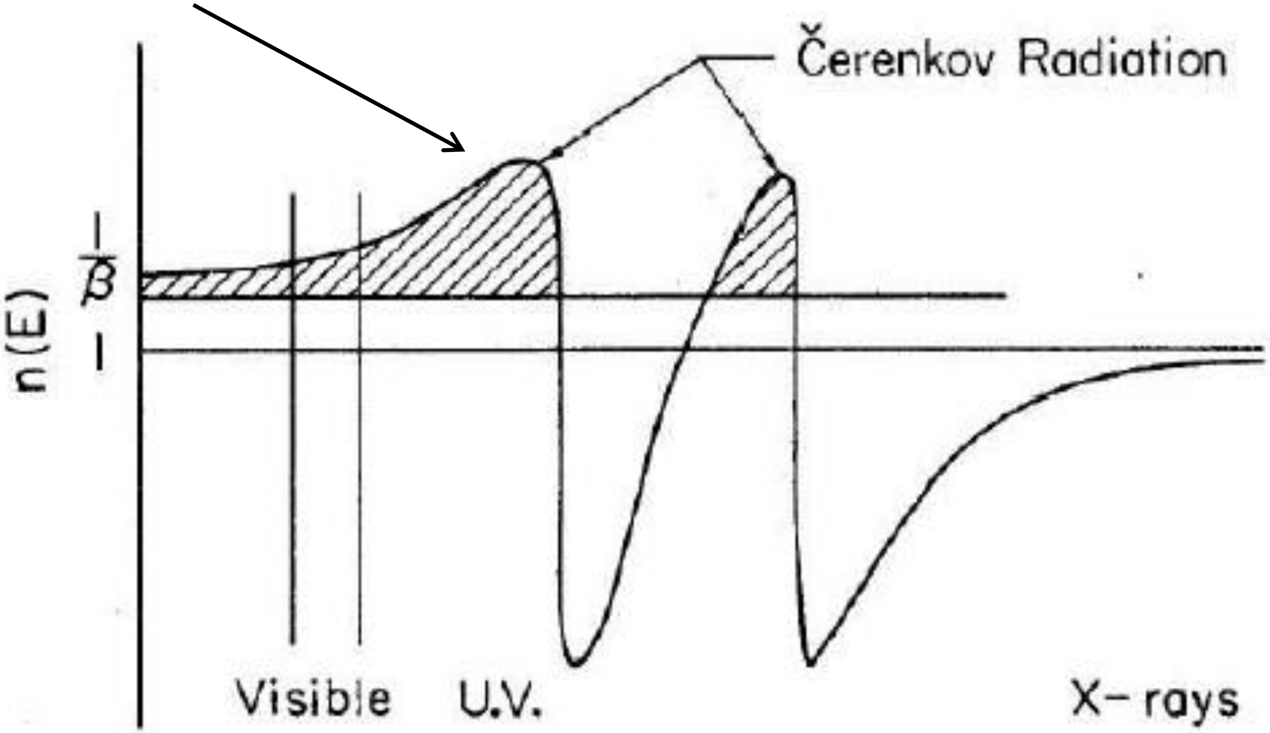
- Previous equation well implies a minimum velocity $v_{min} = c/n$ (and consequently → $n > 1$)
- With $T = E - m_0c^2 = (\gamma - 1)m_0c^2$:

$$T_{min} = m_0 c^2 \left(\frac{n}{\sqrt{n^2 - 1}} - 1 \right)$$

- For electron in water: T_{min} = 264 keV
- For proton in water: T_{min} = 486 MeV
- Cherenkov effect only for incidents electrons (for energies considered here)
- Refractive index varies with wave length → as we need $n(\lambda) > 1$ → only wave lengths for which this condition is fulfilled appears in the emission spectrum→ no X-ray

Refractive index of water

Maximum in the blue



- The medium must be transparent in the visible to allows to detect Cherenkov effect

Contribution to energy loss

- Number of photons emitted per unit length and per frequency unit →

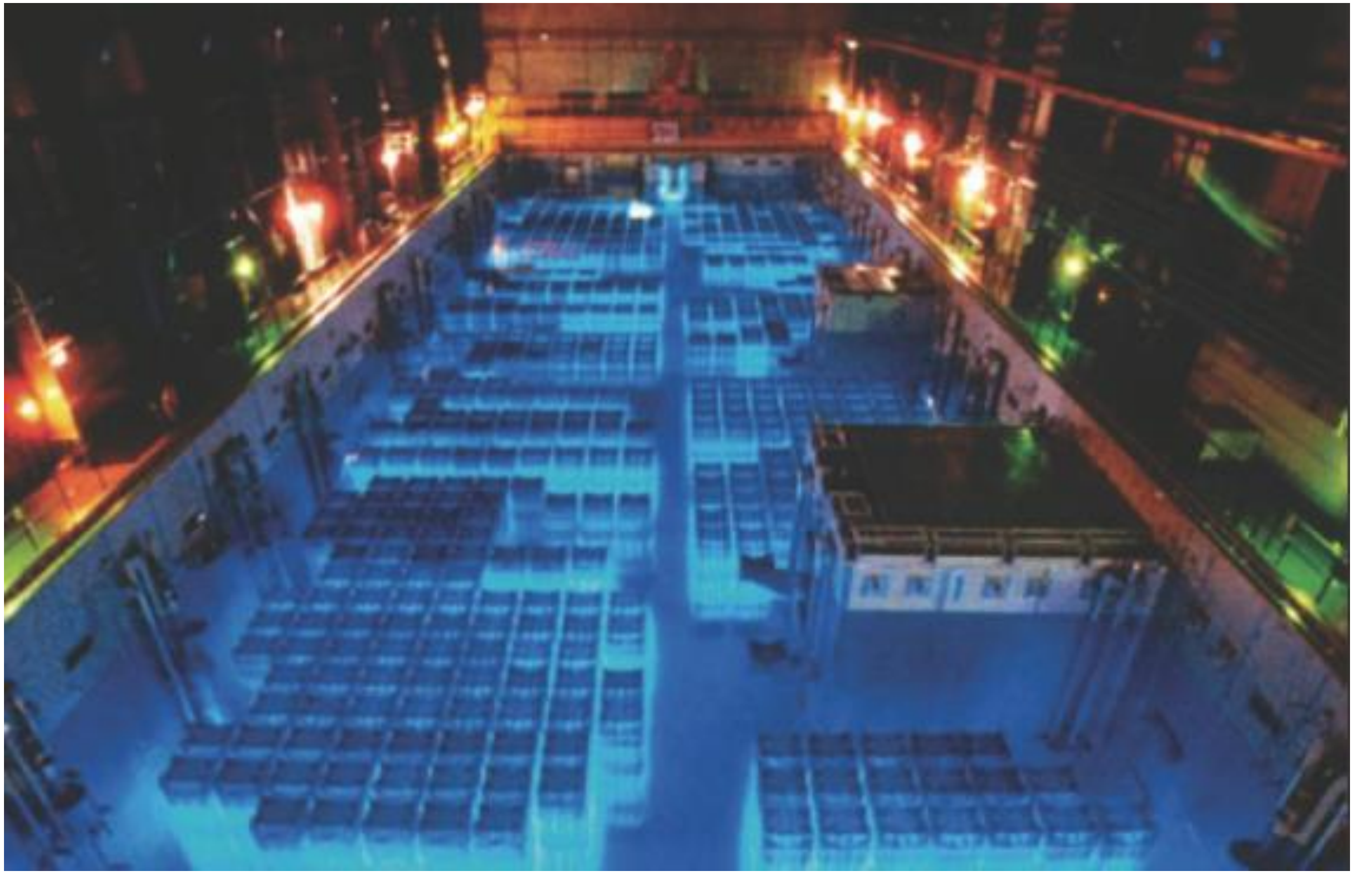
$$\frac{d^2N}{d\nu dx} = \frac{2\pi\alpha z^2}{c} \sin^2\Theta_c$$

- For an electron (*z* = -1) and an optical windows between 350 nm and 500 nm (with n independent on λ in this windows) →

$$\frac{dN}{dx} = 390 \sin^2\Theta_c (\mathrm{cm}^{-1})$$

- Very small number of photons → no contribution to the energy loss

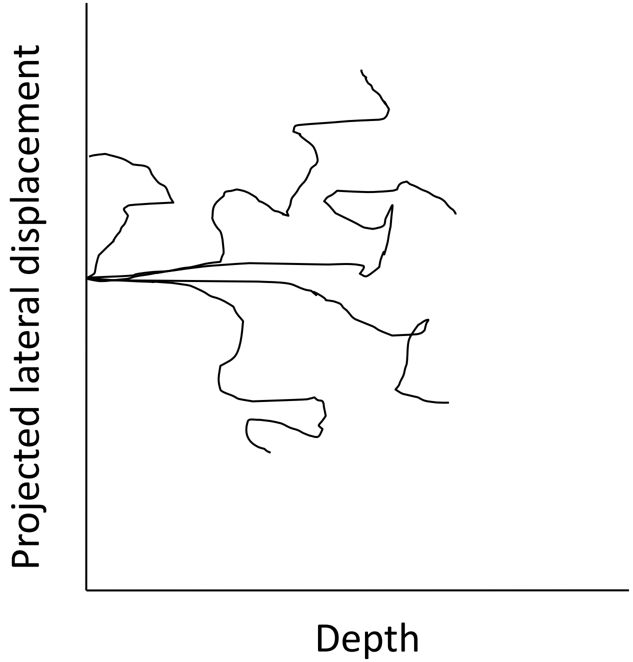
Cherenkov effect: example



Fuel assemblies cool in a water pond at the nuclear complex at La Hague

Electron trajectories (1)

- The notion of range for electrons is not so clear than for ions → the electron trajectory cannot be considered as a straight line → large angular deviations are possible (during electronic and nuclear collisions)



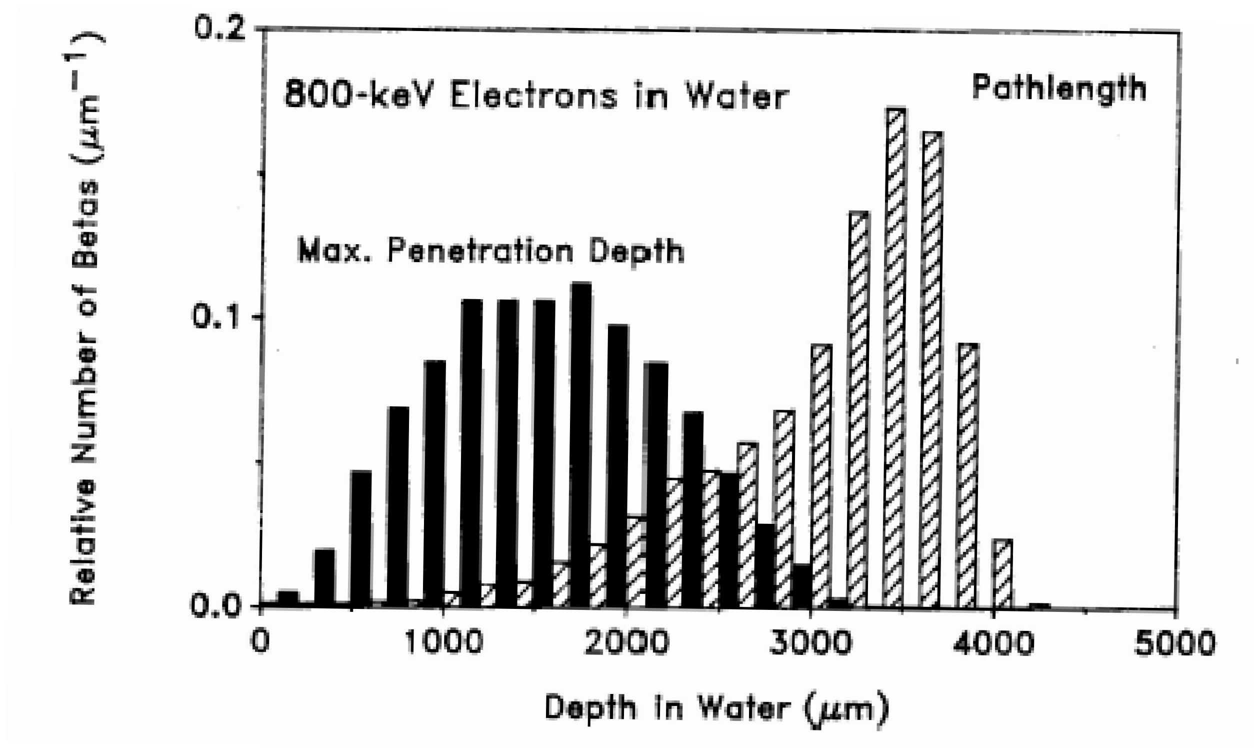
Electron trajectories (2)

- Moreover the electron can lose an important fraction of its energy in only one collision (→ 50%) → depth penetration and length of the trajectory are random with large distributions → important straggling

- In databases → tabulation of the range CSDA, R_{CSDA} → large difference can occur between R_{CSDA} and real range

- Detour factor can be very different from 1 (≈ 0.9 for media with small Z but can reach ≈ 0.5 for large Z)

Depth penetration and trajectory length: Example



Distributions measured for a 800 keV electron in water

Detour factor : Examples

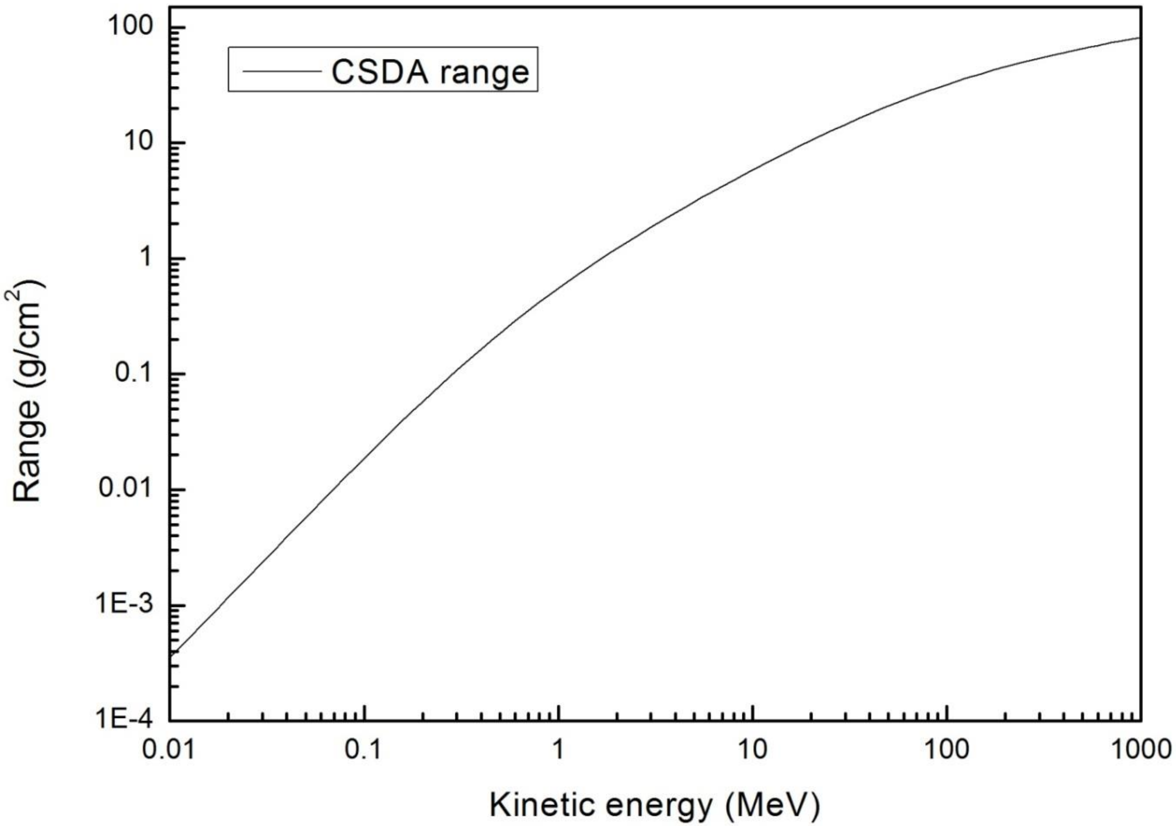
T /MeV	Z	$d_{\max}$ /mg · cm^{-2}	$\mathbb{R}_{\text{csda}}$ /mg · cm^{-2}	$d_{\max}/\mathbb{R}_{\text{csda}}$
0.05	13(Al)	5.05	5.71	0.88
0.10	13(Al)	15.44	18.64	0.83
0.15	13(Al)	31.0	36.4	0.85
0.05	29(Cu)	5.42	6.90	0.79
0.10	29(Cu)	17.1	22.1	0.77
0.15	29(Cu)	34.0	42.8	0.79
0.05	47(Ag)	5.04	7.99	0.63
0.10	47(Ag)	15.6	25.2	0.62
0.15	47(Ag)	30.2	48.4	0.62
0.05	79(Au)	4.73	9.88	0.48
0.10	79(Au)	14.3	30.3	0.47
0.15	79(Au)	27.6	57.5	0.48

CSDA range: Examples (1)

- For 1 MeV electron in lead → R_{CSDA} = 0.7mm
- For 1 MeV electron in silicon → R_{CSDA} = 2mm
- For 1 MeV electron in air → R_{CSDA} = 4076 mm

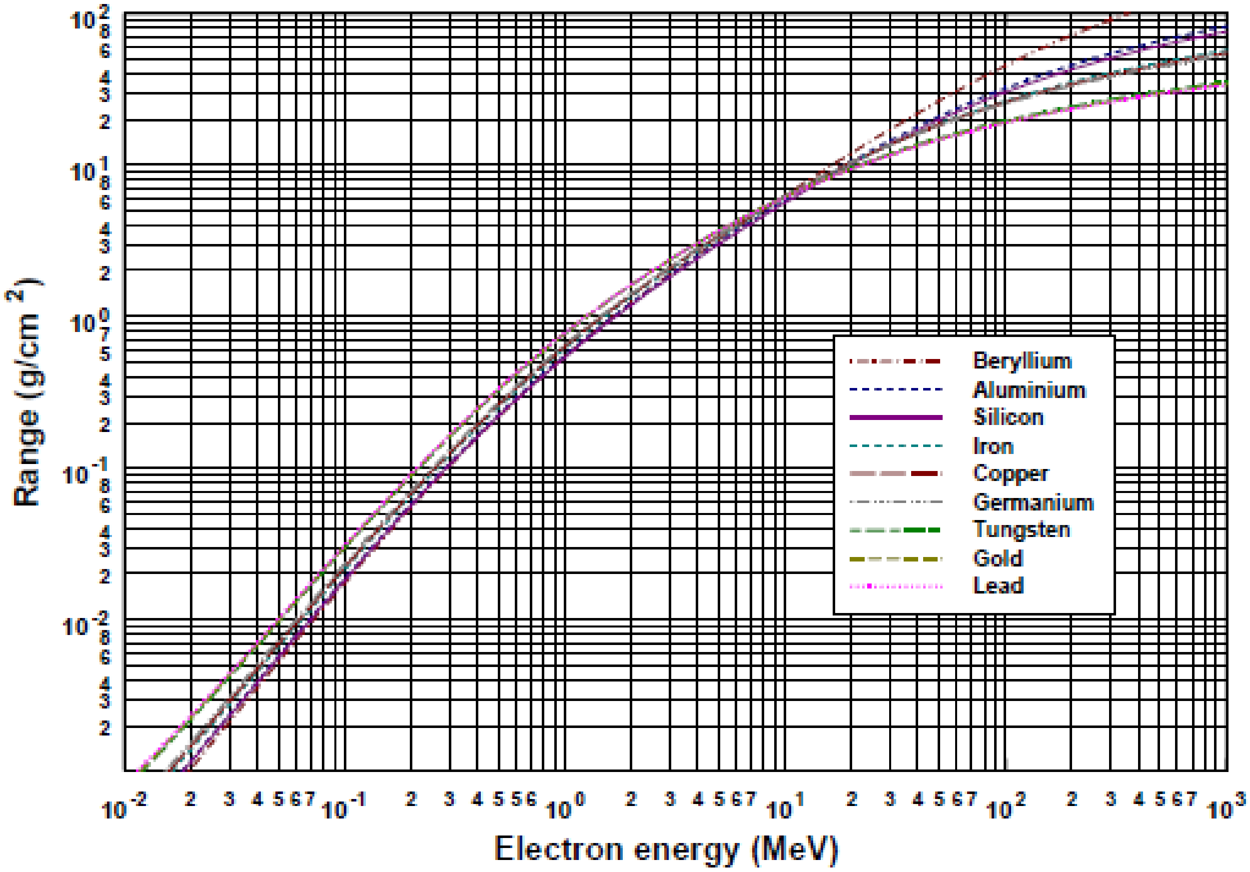
CSDA range: Examples (2)

Incident electron on aluminium (ρ =2.70 g/cm^3)



http://www.nist.gov/pml/data/star/index.cfm

CSDA range: Examples (3)



As for ions → we show $R_{CSDA} \times \rho \rightarrow \rho R_{CSDA}$ (quasi-) independent from medium, especially for small energies

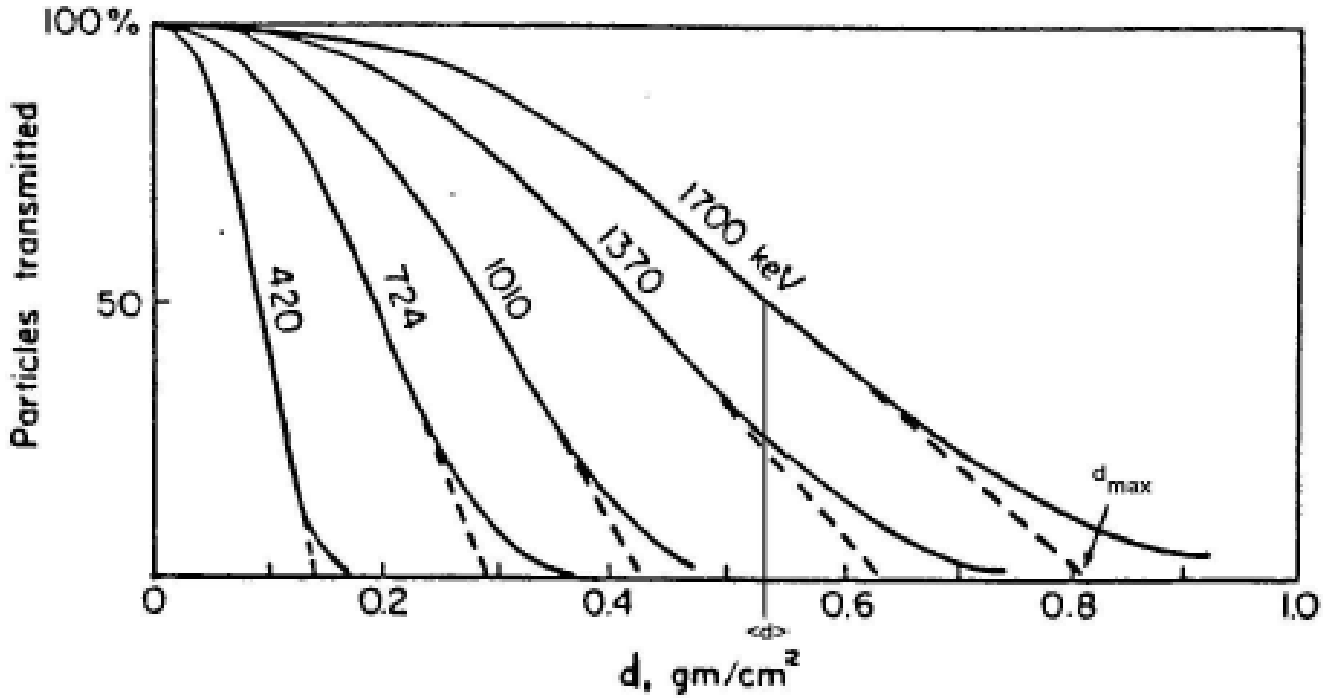
Empirical equation for the range

For media with small Z → empirical equation (with ρR_{CSDA} in gcm^{-2} and E in MeV) →

$$\rho R_{CSDA} = \begin{cases} 0.412 E^{1.27 - 0.0954 \ln E} & \text{for} \quad 0.01 < E < 2.5 \\ 0.530 E - 0.106 & \text{for} \quad E > 2.5 \end{cases}$$

Transmission of electrons

Shape completely ≠ from the shape obtained for ions (rectangle) →



Transmission of electrons β (1)

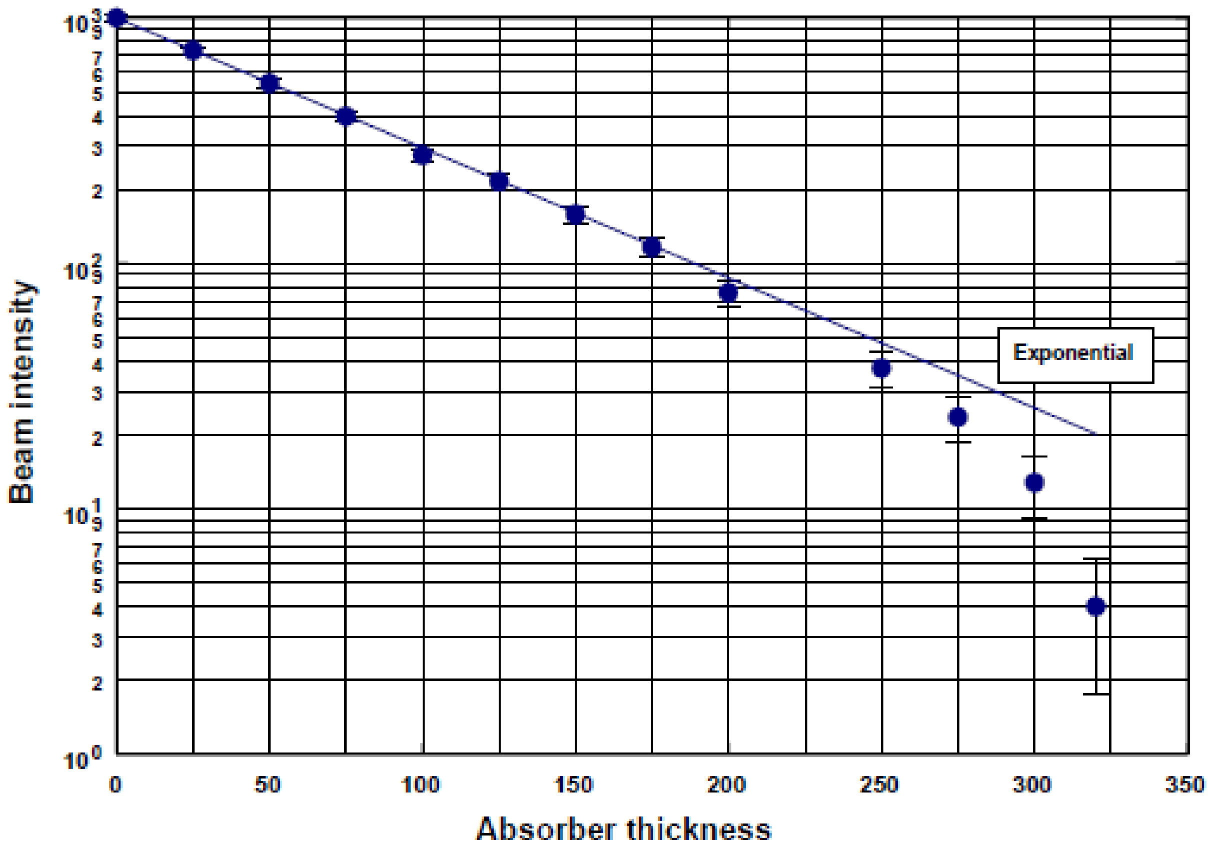
- During disintegration β → the β and a neutrino share the available energy between them → spectrum in energy for the β continuous between 0 and E_{max} → « bell-shaped » curve
- When the β attenuation is observed → behaviour $\sim$ to a decreasing exponential → the ratio of the transmitted intensity I on the initial intensity I_0 is approximated by →

$$\frac{I}{I_0} = \exp\left(-n\rho d\right)$$

- With ρ, the density of the medium, d, its thickness and n, the absorption coefficient i.e. a constant depending on E_{max} (and weakly on the medium) → empirical expression for n (m^2kg^{-1}) →

$$n = 1.7E_{max}^{-1.14}$$

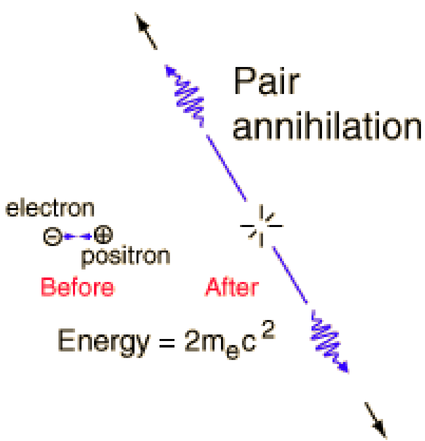
Transmission of electrons β (2)



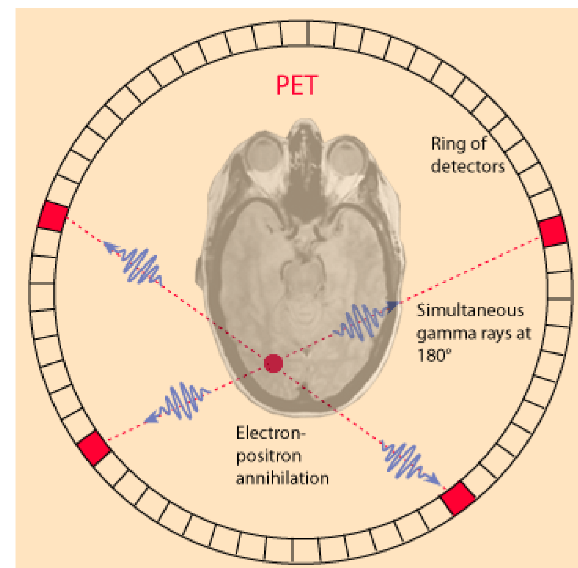
Approximate expression → false for a material thickness $\sim$ to the range of electrons with energy E_{max}

Positron annihilation

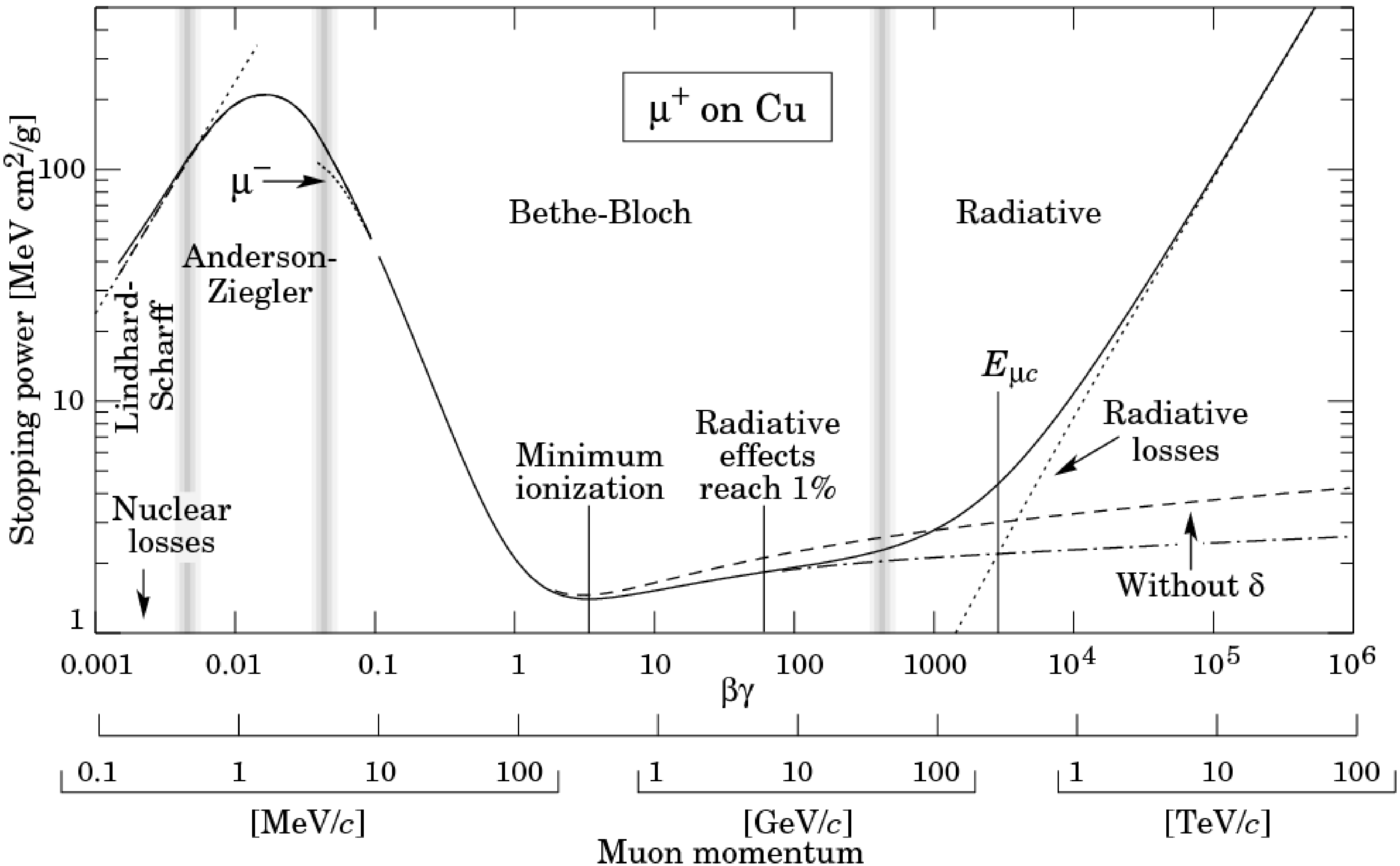
Annihilation of the e^+ after the loss of all its kinetic energy → different processes are possible → the most probable is the annihilation with an e^- at rest → emission of 2 γ of 511 keV each (conservation of energy and momentum)



Application: PET Scan



Example of muon ("heavy electron")



$m(\mu)$=207 $m(e^-)$